HOMEWORK FOR MATH 147 FALL 2024

Problems assigned from Marsden and Weinstein will be preceded by MW. Problems assigned from the Openstax Calculus 3 textbook will be preceded by OS.

Monday, August 26. MW: Section 14.3, #3, 5, 13, 15, 37, 43.

Wednesday, August 28. OS: Section 4.2, # 61, 62, 63, 66, 74, 80, 81.

Friday, August 30. 1. Use polar coordinates to analyze the following limits:

(i) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$.
(ii) $\lim_{(x,y)\to(0,0)} \frac{x^3+x^5}{x^2+y^2}$.

2. Determine the value of the constant c so that $f(x,y) = \begin{cases} \frac{x^3+xy^2+2x^2+2y^2}{x^2+y^2}, \text{ if } (x,y) \neq (0,0) \\ c, \text{ if } (x,y) = (0,0) \end{cases}$ is a continuous function.

3. OS: Section 4.2, $\#102, 110, 111$.

4. For $f(x,y,z) = (x^2+y^2+z^2, 3xyz, \cos(x)+\sin(y)+e^z)$, calculate $\lim_{(x,y,z)\to(1,-1,1)} f(x,y,z)$.

Wednesday, September 4. 1. MW, Section 15.1: # 13-41, every other odd problem, and # 53.

2. For $f(x,y) = \begin{cases} \frac{3x^2y-y^3}{x^2+y^2}, \text{ if } (x,y) \neq (0,0) \\ 0, \text{ if } (x,y) = (0,0), \end{cases}$ find formulas for $f_x(x,y)$ and $f_y(x,y)$.

Friday, September 6. OS, Section 4.4: # 171, 173, 176, as stated, and for each of these problems, find the parametric equations of the tangent lines in the x and y directions at the indicated points. Also: Use the limit definition to show that $f(x,y) = 3x^2 + y$ is differentiable at (1, -1). Then try showing $f(x,y)$ is differentiable at any point (a,b).

Monday, September 9. OS, Section 4.4: # 179, 191, 204, 211.

Wednesday, September 11. OS, Section 4.7: # 311-339, every other odd. Just find the critical points, don't classify them. And: Use an ϵ, δ argument to show that, given a function $f(x,y)$, a point (a,b) in its domain, and $L \in \mathbb{R}^2$, the statements $\lim_{(x,y)\to(a,b)} f(x,y) = L$ and $\lim_{(x,y)\to(a,b)} |f(x,y) - L| = 0$ are equivalent, i.e., each statement implies the other statement.

Friday, September 13. OS, Section 4.7: Classify the critical points you found in # 319-339, every other odd, in the previous assignment.

Monday, September 16. MW, Section 16.3: # 21, 24, 27, 32, 34.

Wednesday, September 18. 1. Find and classify the critical points for:

(i) $f(x,y,z) = x^2 - xy + z^2 - 2xz + 6z$.
(ii) $f(x,y,z) = xy + xz + 2yz + \frac{1}{x}$.
(iii) $f(x,y,z) = e^x(x^2 - y^2 - 2z^2)$.

2. Find the absolute maximum and absolute minimum of $f(x,y,z) = x^2 + xz - y^2 + 2z^2 + xy + 5x$ on the solid block whose coordinates satisfy: $-5 \leq x \leq 0;\ 0 \leq y \leq 3;\ 0 \leq z \leq 2$.

Friday, September 20. OS, Section 4.5: #215, 217, 219, 243, 244, 254.

Monday, September 23. MW, Section 16.1: # 21, 22, 27, 33; Section 16.2, # 21, as stated and also, find the corresponding tangent plane; And: Use the limit definition to find the directional derivative of $f(x,y) = 3x^2 + 2xy + 5$ at (1,2) in the direction of $\cos(\frac{\pi}{3})\vec{i} + \sin(\frac{\pi}{3})\vec{j}$, then verify your answer by direct calculation.

Wednesday, September 25. This homework problem is **extra credit**, to be turned in on Friday, for a maximum of three bonus points. In class we noted that iterated limits need not be equal, for functions of two variables. The failure of the equality of the limits $\lim_{k\to a}\lim_{h\to b} L(h,k)$ and $\lim_{h\to b}\lim_{k\to a} L(h,k)$ for $L(h,k) = \frac{h+k}{h-k}$, is related to the failure of the $\lim_{h,k)\to(0,0} L(h,k)$ to exist. Here is a sufficient condition:

Equality of Iterated Limits. Given $f(x,y)$ and $(a,b)\in\mathbb{R}^2$, If

(i) $\lim_{(x,y)\to(a,b)} f(x,y)$ exits, and
(ii) $\lim_{x\to a} F(x,y)$ exists for fixed y, and
(iii) $\lim_{y\to b} f(x,y)$, exists for fixed x,

then $\lim_{x\to a}\lim_{y\to b} f(x,y) = \lim_{(x,y)\to(a,b)} f(x,y) = \lim_{y\to b}\lim_{x\to a} f(x,y)$.

1. For $f(x,y) = \frac{x^2}{x^2+y^2}$, show that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist, while each of $\lim_{y\to 0}\lim_{x\to 0} f(x,y)$ and $\lim_{x\to 0}\lim_{y\to 0} f(x,y)$ exist, but are not equal.

2. For $f(x,y) = \frac{x^2+y+1}{x+y^2+1}$, show that $\lim_{(x,y)\to(0,0)} f(x,y)$, $\lim_{y\to 0}\lim_{x\to 0} f(x,y)$, $\lim_{x\to 0}\lim_{y\to 0} f(x,y)$ exist and are all equal.

3. For $f(x,y) = \begin{cases} 1, & \text{if } xy\neq 0 \\ 0, & \text{if } xy = 0 \end{cases}$ show that $\lim_{y\to 0}\lim_{x\to 0} f(x,y) = 1 = \lim_{x\to 0}\lim_{y\to 0} f(x,y)$, but $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

Wednesday, October 2. OS, Section 4.8: # 361-366.

Friday, October 4. OS, Section 4.8: # 377, 379, 382, 384, 387

Monday, October 7. OS, Section 5.1: # 13, 19, 21, 25, 53, 37, 30.

Wednesday, October 9. MW, Section 17.2: # 7-17, odd. Bonus Problem 4: Work the following problem for two bonus points. Suppose $a(t)$ is a function of one variable, and $f(x,y) = a(x)a(y)$. Let R denote the square $[c,d]\times[c,d]$. Prove that $\int\int_R f(x,y)\ dA = (\int_c^d a(x)\ dx)^2$. Due Friday, October 11.

Friday, October 11. OS, Section 5.3: # 149,154,155, 158, 159.

Wednesday, October 16. OS, Section 5.7: # 388, 389, 392, 398.

Friday, October 18. OS, Section 5.7: # 390, 394, 397, 398.

Monday, October 21. Calculate the following improper integrals.

(i) $\int\int_D \frac{1}{\sqrt{xy}}\ dA$, for $D = [0,1]\times[0,1]$.
(ii) $\int\int_D \ln\sqrt{x^2+y^2}\ dA$, for $D = 0\le x^2+y^2\le 1$.
(iii) $\int\int_D \frac{1}{x^2y^3}\ dA$, for $D = [1,\infty]\times[1,\infty]$.

Wednesday, October 23. MW, Section 17.4: # 1, 5, 9, 16.

Friday, October 25. OS, Section 5.4: #193, 914, 211, 212, 231.

Monday, October 28. OS, Section 5.5: # 269, 270, 271.

Wednesday, October 30. OS, Section 5.5: # 253-256.

Wednesday, November 6. OS, Section 3.2:# 41-55, odd and OS, Section 3.3: # 102, 106, 107, 110.

Friday, November 8. MW, Section 18.1: # 29, 30, 32, 34.

Monday, November 11. OS, Section 6.2: # 56, 58, 63, 64.

Wednesday, November 13. OS, Section 6.6: # 273, 276, 277 and the following problem. Use the surface area formula to find the surface area of an open cylinder whose base has radius r and whose height is h. What is the surface area if the cylinder is closed, i.e., has a top and bottom?

Friday, November 15. OS, Section 6.6: # 291,292, 295, 296.

Monday, November 18. OS Section 6.2: # 68, 69, 70 and OS Section 6.6: # 303, 304, 309.

Wednesday, November 20. OS Section 6.2: # 51, 52, 55 and Section 6.6: # 284, 285, 286. For the problems in Section 6.2, the work done by a vector field moving a particle along the curve C is $\int_C \mathbf{F} \cdot d\mathbf{r}$.